Coherent transfer of an optical carrier over 251 km

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We transfer an optical frequency over 251 km of optical fiber with a residual instability of 6×10^{-19} at 100 s. This instability and the associated timing jitter are limited fundamentally by the noise on the optical fiber and the link length. We give a simple expression for calculating the achievable instability and jitter over a fiber link. Transfer of optical stability over this long distance requires a highly coherent optical source, provided here by a cw fiber laser locked to a high finesse optical cavity. A sufficient optical carrier signal is delivered to the remote fiber end by incorporating two-way, in-line erbium-doped fiber amplifiers to balance the 62 dB link loss.

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Recently, there have been significant efforts at transporting stable, coherent signals across fiber-optic networks to compare remotely located atomic clocks and to distribute highly coherent stable signals for different scientific applications [1–4]. The challenge in distributing the signal over an optical fiber network lies in compensating for the fiber-induced phase noise; small temperature changes and vibrations cause the fiber link length to change, thereby causing a Doppler shift on the transmitted signal that must be canceled [5–7].

In one approach, the desired frequency is transported in the microwave domain by amplitude modulation of an optical carrier [3,4]. However, the highest stabilities and lowest timing jitter are achieved using the frequency of the 1550 nm optical carrier itself. At the remote end, the optical carrier can be converted to the desired optical or microwave frequency with a fiber frequency comb [8]. Such a coherent fiber ring network was recently demonstrated with subfemtosecond timing jitter and frequency instabilities below 10^{-18} but over only a limited distance [1]. Recently, Grosche *et al.* transferred an optical carrier over 211 km including 86 km of deployed fiber [2]. Foreman *et al.* have improved earlier results on a transfer over the link between the National Institute of Standard and Technology (NIST) and JILA [7], demonstrating very high stabilities [9]. Here we transfer an optical carrier over a 251 km link that includes 76 km of installed fiber from the Boulder Research and Administration Network (BRAN), 175 km of spooled fiber, and four in-line erbium-doped fiber amplifiers (EDFAs). At 251 km and residual instabilities of 3×10^{-16} at 1 s, our results represent record performance in terms of the coherent transfer of a carrier over long distance.

Despite the significant previous work in this area, the very basic question of how far a coherent signal can be successfully transported has never been addressed. While it has been recognized that the delay caused by the length of the link limits the feedback bandwidth for the fiber-noise cancellation, its effect is actually much more pernicious. The basic Doppler cancellation technique relies on suppressing the phase noise on the round-trip light with a phase-locked loop (PLL), under the assumption that the noise on the one-way light will be similarly suppressed. However, even assuming *perfect* cancellation of the noise on the round-trip signal, the delay through the link results in the *imperfect* cancellation of the noise on the one-way signal.

Empirically, we find our link exhibits a phase noise with the power-law dependence on Fourier frequency, f, of $S_{\mathrm{Fiber}}(f)\sim h/f^2$, up to a cutoff frequency, $f_c \sim 1$ kHz, beyond which the noise drops more rapidly. (The magnitude of the noise, h, will presumably vary for different links and equals $\sim10^3\ \mathrm{rad}^2/\mathrm{Hz}$ here.) As derived later, even for a perfect round-trip phase lock, this fiber noise is not completely canceled for the one-way light. Instead, the one-way phase noise is at best $a(2\pi f\tau)^2 S_{\mathrm{fiber}}$, where $a\sim1/3$ and τ is the time for the signal to travel the fiber link length L, i.e., the delay. The resulting white phase noise of $a(2\pi\tau)^2 h$ causes a fractional frequency uncertainty of

$$\sigma_\nu = \nu^{-1}\sqrt{8ah}\,\pi\tau t_{\mathrm{gate}}^{-3/2}, \qquad (1)$$

on the delivered optical carrier ν, measured with a typical high-resolution "Λ-type" frequency counter [10], where t_{gate} is the gate time. Since τ and h are both proportional to the fiber length L, the uncertainty grows rapidly as $L^{3/2}$ and drops similarly rapidly with t_{gate}.

Similarly, the resulting timing jitter [i.e., $(2\pi\nu)^{-1}$ times the integrated phase noise] is limited to

$$t_{\mathrm{jitter}} \approx \nu^{-1}\sqrt{ahf_c}\,\tau, \qquad (2)$$

assuming the effective feedback bandwidth of the round-trip noise cancellation f_{fb} exceeds the noise cutoff frequency, f_c (i.e., a perfect round-trip phase lock). The feedback bandwidth is limited to $f_{fb} < 1/(4\tau)$, because of phase shifts (although a higher f_{fb} is achievable with an appropriate servo filter [11]). Beyond $\sim$50 km, $f_{fb} \not\geq f_c$ and t_{jitter} can exceed Eq. (2) due to additional uncorrected phase noise.

The incomplete suppression of the one-way fiber noise due to the delay dominates other limitations. The signal-to-noise ratio (SNR) on the round-trip signal, $\mathrm{SNR}_{\mathrm{RT}}$, is an obvious concern. Indeed, without amplification, this SNR decays exponentially with length as $\mathrm{SNR}_{\mathrm{RT}}=\eta e^{-2\alpha L}\mathrm{SNR}_{\mathrm{input}}$, where $\mathrm{SNR}_{\mathrm{input}}$ is the shot noise on the launched input power of a few milliwatts, limited by stimulated Brillouin scattering (SBS), η is the detection efficiency, and $\alpha \sim 0.2$ dB/km is the fiber loss. Beyond $L=150$ km, the round-trip signal drops below 1 nW, making a simple phase lock difficult. However, as is well known, EDFAs can boost the signal and SNR. For a transparent network (link gain equals link loss) with k amplifiers, each with a noise figure F, $\mathrm{SNR}_{\mathrm{RT}} \approx \eta(2kF)^{-1}\mathrm{SNR}_{\mathrm{input}}$ [12]. For a 100 km amplifier spacing (gain=20 dB and $F\sim 4$), the SNR is still sufficient (>90 dB/Hz) for $L>10^7$ km; therefore, the SNR is not a practical limit.

Figure 1 shows the schematic of our setup, which uses the established Doppler cancellation technique [5–7]. To generate a source with a coherence length exceeding the round-trip length, a 50 mW cw fiber laser is phase locked to a stable optical cavity (finesse is 165,000, linewidth is 9 kHz) with the Pound–Drever–Hall technique. The laser has a linewidth of ~1 Hz and a frequency drift of a few hertz per second, as measured with a fiber frequency comb [13]. Part of this light serves as a local oscillator (LO), while the remainder is launched through an acousto-optic modulator (AOM) into the fiber link, ultimately circling back to the same laboratory at the far end of the link. At this end, half of the output one-way light is heterodyned against the LO and the resulting frequency is analyzed for stability and timing jitter. The other half is retroreflected back to the start of the fiber link, where it is also heterodyned against the LO. A PLL locks this round-trip signal to an rf source by serving back to the input AOM. A second AOM at the far end shifts the true round-trip reflected signal to distinguish it from other intermediate reflections. Periodic (~hourly) polarization adjustment maximized the SNR. Maximum EDFA gains are limited by SBS and lasing thresholds.

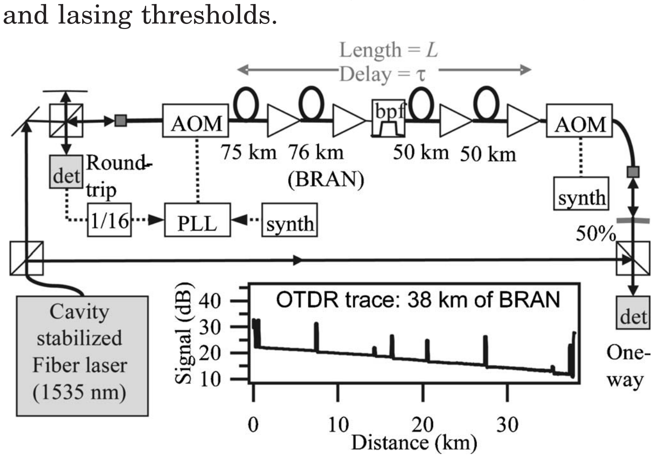

Fig. 1. System layout. Heavy solid lines are fiber paths, thin lines are free-space optical paths, and dotted lines are electrical paths. AOM, acousto-optic modulator; triangles, EDFAs; bpf, bandpass filter at 1535 to reduce amplified spontaneous emission (ASE) buildup; 50%, partially reflecting Faraday mirror. Inset, optical time-domain reflectometry (OTDR) trace of one 38 km traverse of the BRAN. Each peak represents a location where the fiber came above ground into a building with a patch panel.

The fiber link uses the installed BRAN with light traversing the city of Boulder twice (via parallel fibers) for a total of 76 km. We speculate that this fiber is particularly noisy (20–30 dB more than an equivalent length of spooled fiber) because of its 16 excursions to above ground patch panels and ~50 fiber connectors (Fig. 1). To extend the link, 75 km of spooled fiber is added prior to the BRAN and 100 km after, with four in-line EDFAs to overcome the 124 dB round-trip link loss from fiber alone (excluding loss from the AOMs and fiber launches).

Figure 2 shows the measured phase noise from the fiber, which fits empirically to $S_{\mathrm{fiber}}\sim 10^3/f^2$ until a cutoff frequency of $f_c\sim 1$ kHz, whereupon the phase noise falls off more rapidly. The laser phase noise is sufficiently low that its contribution to S_{fiber} due to the delayed self-heterodyne effect is $<10^{-5}$ rad^2/Hz even at $L=251$ km [13]. From feedback theory, the phase noise on the locked round-trip signal is $S_{\mathrm{Locked,RT}}\sim|G(f)|^{-2}f^2 S_{\mathrm{Fiber,RT}}$ within the feedback bandwidth, $f_{fb}\sim 1/(4\tau)$, where $G(f)$ is the loop gain, and $S_{\mathrm{Fiber,RT}}\sim 4S_{\mathrm{fiber}}$. Above 0.1 Hz, the PLL gain is proportional and for our settings, $S_{\mathrm{Locked,RT}}\sim 2\times 10^{-7}f^2 S_{\mathrm{Fiber,RT}}$.

In contrast, as shown in Fig. 2, the locked noise on the one-way signal, S_{Locked}, exceeds $S_{\mathrm{Locked,RT}}$ by ~20 dB for the reasons given here. Consider a phase perturbation to the signal, $\delta\varphi_z(t)$ at a distance z from the fiber start. The perturbed phase of the round-trip light is $\delta\varphi_{\mathrm{Fiber,RT}}(t)=\delta\varphi_z(t-z/v)+\delta\varphi_z(t-2\tau+z/v)\sim 2\delta\varphi_z(t)-2\tau\delta\varphi_z'(t)$, where $\varphi_z'=\mathrm{d}\varphi_z/\mathrm{d}t$ and v is the phase velocity in the fiber. If the PLL is "perfect," a phase correction ϕ_c is applied (via the AOM) so $0=\delta\varphi_{\mathrm{Fiber,RT}}(t)-\{\varphi_c(t)+\varphi_c(t-2\tau)\}$ or $\varphi_c(t)\sim\delta\varphi_z(t)$. The phase perturbation for the one-way light is $\delta\varphi_z(t-\tau+z/v)-\varphi_c(t-\tau)\approx(z/v)\delta\varphi_z'$ and *does not vanish* (unless in steady state or at $z=0$). A Fourier transformation, denoted by a tilde, and integration over the fiber

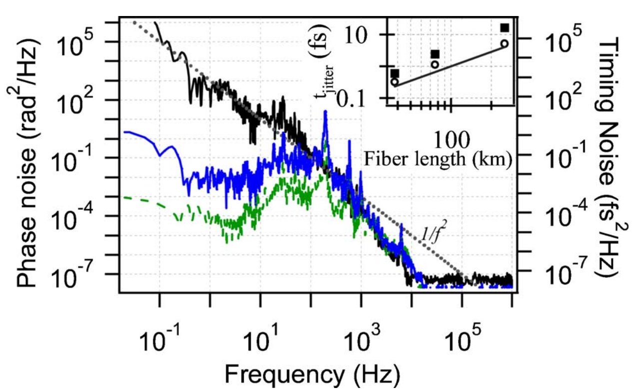

Fig. 2. (Color online) Residual phase noise (left axis) and timing noise (right axis) of the fiber link measured on the unlocked one-way signal, S_{fiber} (solid, black line); the one-way locked signal, S_{Locked} (solid, blue curve); and the round-trip locked signal, $S_{\mathrm{Locked,RT}}$ (dashed, green curve) for the 251 km link. The fiber noise falls off as $1/f^2$ (dotted curve) up to ~1 kHz. The spikes on the locked noise occur at harmonics of $f=1/(4\tau)=210$ Hz. Inset, measured timing jitter with (solid squares), and without (open circles), the contribution from servo spikes, and the predicted jitter from Eq. (2) (solid curve).

length yields $S_{\text{Locked}} \approx |\int_0^L (z/v)^2 2\pi i f \delta\tilde{\varphi}_z(f) dz|^2 = a(2\pi f \tau)^2 S_{\text{fiber}}$, where $S_{\text{fiber}} = \int_0^L |\delta\tilde{\varphi}_z(f)|^2 dz$ and $a = 1/3$ if $|\delta\tilde{\varphi}_z|^2$ is independent of z. For our link, $\tau = 1.2$ ms and so $S_{\text{Locked}} \approx a(2\pi f\tau)^2 S_{\text{fiber}} \approx 2.7 \times 10^{-5} f^2 S_{\text{fiber}}$, which prediction falls exactly on the measured curve, 20 dB higher than the locked round-trip phase noise. In so much as $S_{\text{fiber}} = h/f^2$, S_{Locked} is white phase noise given simply by $S_{\text{Locked}} \approx (2\pi\tau)^2 ah$.

The integrated timing jitter, Eq. (2), follows directly from this white phase noise approximation. The inset of Fig. 2 compares the measured and predicted timing jitter from Eq. (2) at different link lengths (measured by removing spools of fiber). The timing jitter is ~16 fs at 251 km, if the servo spikes are included, but drops close to the 4 fs prediction of Eq. (2) without their contribution. As expected, we observe a coherent peak on the rf power spectrum of the one-way signal up to $L \sim 100$ km, beyond which the integrated phase jitter exceeds ~1 rad.

The predicted frequency uncertainty, σ_ν, of Eq. (1) also follows directly from the white phase noise approximation for the locked one-way phase noise. To measure σ_ν, we counted the one-way and round-trip heterodyne beat signals. For $t_{\text{gate}} < 10$ s the uncertainty is the standard deviation frequency counter readings, and for $t_{\text{gate}} > 10$ s the uncertainty is the modified Allan deviation calculated from contiguous counter readings [10]. The modified Allan deviation was used for two reasons. First, it is the appropriate deviation for white phase noise. Second, the high-resolution frequency counters used here, and elsewhere at NIST, actually report a quantity similar to the modified Allan deviation and not the regular Allan deviation. For details, see [10]. As predicted from Eq. (1) for $h \sim 10^3$ (rad^2 Hz) and $\tau = 1.2$ ms, $\sigma_\nu = 3 \times 10^{-16}$ at 1 s, falling off as $t_{\text{gate}}^{-3/2}$ until it reaches the system noise floor. (The regular Allan deviation calculated from a contiguous set of counter readings taken at 1 s would instead fall off as $3 \times 10^{-16} t_{\text{gate}}^{-1/2}$.)

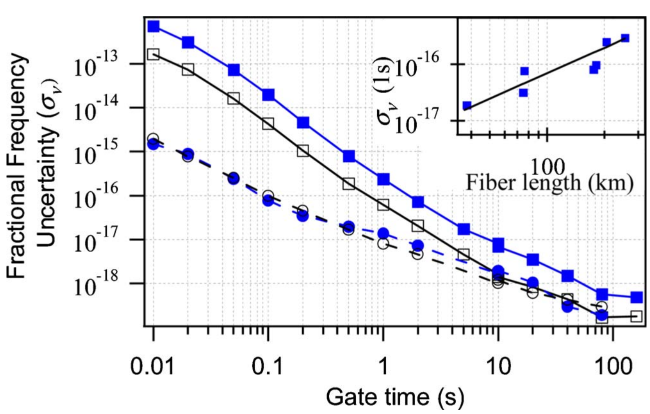

Fig. 3. (Color online) Residual fractional frequency uncertainty for the one-way (solid squares) and round-trip (open squares) signals over the 251 km link, and the system noise floor for the one-way (solid circles) and round-trip (open circles) as measured for a 0 km link. Inset, measured (solid squares) and predicted (line) uncertainty versus link length.

This low system noise floor requires a compact, free-space interferometer; a fiber-based system had significantly higher noise due to out-of-loop fiber. We observe no polarization-mode dispersion or other effects causing a flicker noise floor. Assuming h is dominated by the BRAN fiber, Eq. (1) predicts a linear scaling of σ_ν, as shown in the inset of Fig. 3.

Even at 251 km, the residual instability of the transferred signal is below that of current optical clocks, and longer links should be possible, limited in practice by EDFA saturation from multiple reflections and ASE buildup. Based on this work, long-distance high-stability frequency transfer is possible, provided the installed fiber connecting the sites allows for two-way propagation.

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